to realize a payoff even in rather severe hostile environments. Further, it appears that defensive systems with weights above about 20% of the mission payload will pay off only in the most severe environments ($a_0 > 0.2$) and then only if costs are moderate. The cross plots in Fig. 4 tend to bear out the foregoing conclusions in a different manner. For the severe operational environment case ($a_0 = 0.50$) it appears that a penetration aid offering an improvement better than 2:1 in survivability will pay off if its costs are less than about 10% of the combined mission equipment and total weights are less then 10% of the combined mission payload. Yet, where nominal attrition ratios are expected to be lower ($a_0 = 0.10$), either the improvement from using penetration aids must be high or the costs and weights must be very low in order for a benefit to be realized.

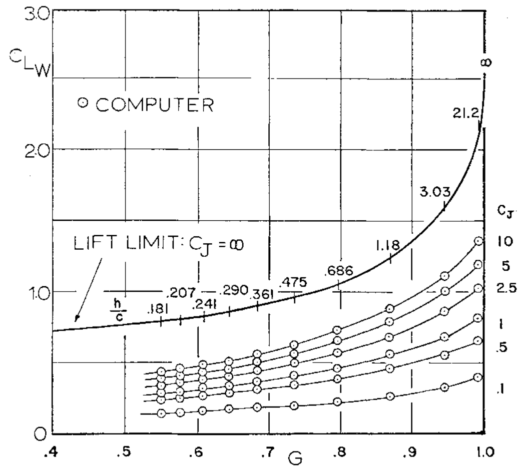

Fig. 2 Wake lift component.

Design Data for the Jet Flap in Ground Effect

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Introduction

THE jet flap is of considerable interest for obtaining very high lifts from an airfoil. This is significant for STOL, for helicopters, and for fluid support of high-speed ground vehicles, or for amplification of conventional flap-type controls. Linearized solutions for the two-dimensional case in an infinite fluid are well-known (for example, Spence[1]). The presence of the ground causes significant effects on a jet flap airfoil, due both to ground-induced pressure and viscous ground effects (blockage). This note distils the linear theory solution for the jet flap in ground effect. The analysis is described in detail in Refs. 2 and 3; here only those results of direct engineering interest are given for use in design.

Theoretical Development

The two-dimensional, inviscid, incompressible steady case is considered (Fig. 1). The problem is to find a potential function, Laplacian in the general field, which satisfies normal boundary conditions on the airfoil, ground plane, and at infinity, and an additional condition on the jet wake, namely no flow through the wake, with the pressure difference across the wake related to the local curvature and the jet blowing coefficient $C_J = J/qc$. Here J is the jet momentum, q the freestream dynamic pressure, and c the chord. Although the field equation is linear, the boundary conditions are not, since on the wake the dynamic condition implies matching terms of order V^2 while the wake shape itself changes under different lifting conditions.

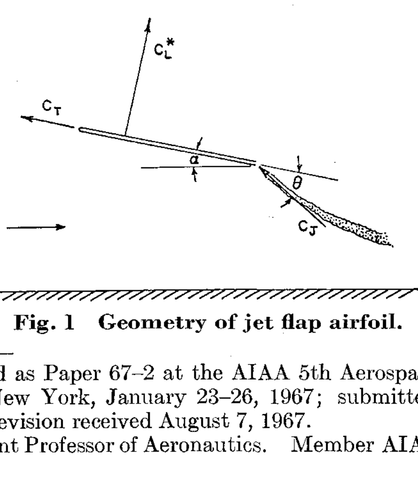

Fig. 1 Geometry of jet flap airfoil.

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The problem is linearized in the usual thin airfoil fashion, and the boundary conditions applied on some mean line. This still does not explicitly define the boundary condition on the jet wake, but transforms the problem to a linear one with Riemann-Hilbert-Poincare boundary conditions.[3] It may now be considered as four distinct superimposable cases: the classical ones of thickness, camber, and angle of attack (α), and the additional case for variation of the jet efflux angle (θ) (the singular problem). These solutions are functions of C_J and h/c, h being the height of the airfoil.

Details of the technique of solution are not discussed; however, the nose flow is of especial engineering interest. For the case of zero thickness and camber (Fig. 1), where C_L^* is the coefficient of pressure lift on the upper and lower surfaces only, and C_T the nose thrust coefficient, C_L^* may be eliminated from the two equations of vertical and horizontal equilibrium. The lift coefficient C_L is linear in α and θ. After a further elimination we get

$$C_T = C_J(\theta^2/2) \qquad \alpha = 0 \tag{1}$$

$$C_{L\theta}{}^2 = 2C_J C_{L\alpha} - C_J{}^2 \tag{2}$$

Equation (1) gives a direct insight into the thrust recovery mechanism. For the case $\alpha = 0$, the jet momentum thrust at the trailing edge is $C_J(1 - \theta^2/2 + \ldots)$; it is now clear that the nose suction provides the additional force on the airfoil for full thrust recovery. Physically, the deflection of the jet induces a flow around the nose of a related magnitude. Thus for practical designs attention must be given to the control of the nose flow to realize thrust recovery. Equation (2) shows that it is unnecessary to solve both the angle-of-attack case (for $C_{L\alpha}$) and the singular case (for $C_{L\theta}$) since they are directly related.

Lift of the Airfoil

Having solved the boundary value problem, the pressure distribution may be determined. This is singular, $O(x^{-1/2})$ and $O(C_J \log|c - x|)$, at the leading edge ($x = 0$) and trailing edge ($x = c$). We can isolate these singularities, and express the lift as the sum of three terms. The first, the nose lift C_{LN}, is principally due to pressures near the nose, and appears in closed form as $C_{LN} = 4\pi^{-1/2}C_J{}^{1/2}G^{-1}$. Here G is the geometrical parameter describing the height of the airfoil. For engineering purposes h/c as a function of G may be read off from Fig. 2. Note that $h/c \to 0$, $G \to 0$; $h/c \to \infty$, $G \to 1$. The second component is due directly to the jet momentum, given in linear theory as C_J. The final component (mainly due to pressure near the trailing edge)

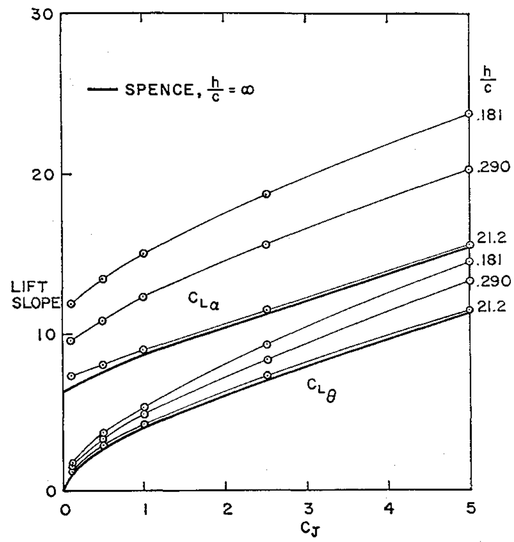

Fig. 3 Lift curves in ground effect.

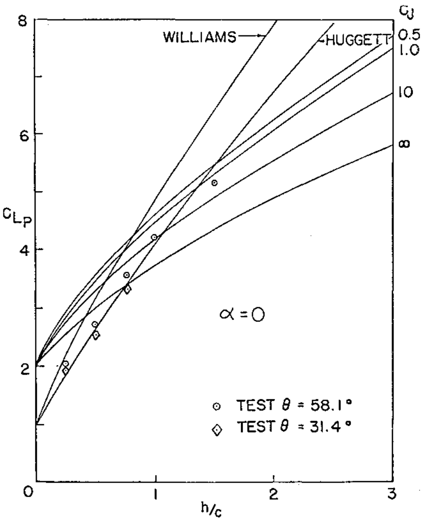

Fig. 4 Wake blockage.

may be transformed into an integral over the jet wake and is thus called the wake lift component C_{Lw}. This is evaluated numerically and is plotted in Fig. 2. We find that C_{Lw} is bounded for $C_J = \infty$ (shown in Fig. 2 as "Lift Limit"). However, for the case of $G \to 0$ and C_J finite we get $C_{Lw} \to 0$.

Thus, we express the singular blowing lift slope as

$$C_{L_\theta} = C_{LN} + C_J + C_{Lw} \tag{3}$$

By using Eq. (2), C_{L_α} may be determined, and is shown in Fig. 3. As a limiting case check, the results of Spence[1] for $h/c = \infty$ are shown.

Interesting results occur for extreme C_J, G values. Pressure lift on the airfoil, $C_{LP} = C_{LN} + C_{Lw}$, is most significant, since the jet momentum lift C_J is unaffected by the ground plane. For large values of h/c and strong blowing, $C_{LN} \sim O(C_J^{1/2}/G)$, $C_{Lw} \sim O(1)$. Thus a dominant part of the lift is carried near the nose. For weak blowing,[3] we get $C_{LN} \sim O(C_J^{1/2}/G)$, $C_{Lw} \sim O(C_J^{1/2})$. Again, for low values of h/c and finite blowing we get $C_{LN} \sim O(C_J^{1/2}/G)$, $C_{Lw} \to 0$. Other results may be obtained similarly, giving an engineering insight into the behavior of the lifting terms. Frequently the nose flow is the critical factor, since high negative pressures and adverse gradients cause both compressible and viscous effects. This may be controlled by detail design of nose radius and droop, involving thickness and camber effects for which a procedure is given in Ref. 3.

Wake Blockage

This phenomenon is important since it may make it impossible to achieve the theoretical lift. Experiments by Huggett[4] showed that blockage caused a behavior analogous to conventional stall, where dC_L/dC_J abruptly reduces and the lift on the airfoil reaches a maximum. This is caused by the jet impinging on the ground and effectively blocking the lower surface flow. For a given h/c, the pressure lift (C_{Lp}) remains constant for all C_J above a critical value. Tests showed this critical C_{Lp} was independent of θ ,for $\alpha = 0$.

Theoretical estimates of the maximum C_{Lp} are made by Williams[3] and Huggett.[4] Details of both models can be criticized. An independent estimate of using the present linear theory gives a formula for blockage as $C_{Lp} = (2h/c)^{1/2}\{4\pi^{-1/2}/G + C_{Lw}C_J^{-1/2}\}$. This curve has similar characteristics to those of Ref. 4, although it too is certainly not a precise model of the flow. The three curves are shown in Fig. 4, with some data.[4] The tests of Ref. 4 were with a fixed ground although these are improper boundary conditions.

Evidently a true test involves removing the ground boundary layer, by suction or a moving belt. This might be expected to increase the value of blockage C_{Lp}. None of the three blockage theories takes explicit account of viscosity. Consequently, both the theoretical and experimental values of Fig. 4 should be regarded as very approximate; however, in practical design situations blockage will be a major consideration.

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Redundant Analysis of a Group of Airframe Problems

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A LARGE number of the mathematical models prepared for discrete-element redundant analysis of airplane structural problems at the Lockheed-Georgia Company since November 1961 were reviewed recently. A comparison was made of the maximum number of simultaneous equations to be solved by application of the force and the displacement methods of analysis for compatibility or equilibrium, respectively. The results of the review are shown in Table 1 as the number of redundants n for the force method and the degrees of freedom d for the displacement method.

The tabulation shows that the maximum number of simultaneous equations to be solved for displacement-method application is three to five times more than for the force-method application to the practical airframe design engineering problems reviewed. Table 1 is representative of the redundant-analysis problem solutions performed for the C-141 and C-5A airplanes. All of the mathematical models listed in Table 1 were solved by the particular version of the force method, which is described in Ref. 1. Computations were performed on IBM 7094 computers via the flutter and matrix algebra

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